

Ordering and comparing fractions

Key National Curriculum statement

Recognise and show, using diagrams, equivalent fractions with small denominators
 Recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators
 Compare and order unit fractions and fractions with the same denominators

Related National Curriculum statements

- count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10
- recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators
- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables

Key concepts

This sequence builds upon learning from 3LS21 in which pupils were introduced to unit and non-unit fractions, tenths as 10 equal parts and explored different representations to find and identify fractions of discrete sets of objects and continuous quantities.

Pupils build on their prior experiences to find fractions of regular and irregular shapes. At this stage, all of the parts are congruent.

Steps 2 and 3 are focused upon comparing and ordering fractions. First, focusing on unit fractions (fractions with a numerator of 1) and then fractions with the same denominator. In these steps, pupils require clarification about the relationship between the size of the parts and the whole. For example, the larger the denominator, the smaller the part - the whole has been divided into more equal parts. Teachers should refer to the relationships between parts and wholes frequently.

Later in the sequence, pupils will build upon their understanding of the equivalence of one half and two quarters, which is first introduced in year 2. Ranges of representations are used to allow pupils to fully explore equivalence. Pupils create their own diagrams, make sure that the wholes are the same size / quantity and that they are comparing the number of parts and the size of the parts.

Curriculum progression: tracking back

2LS29

1LS33

Steps within the Learning Sequence

Step 1: Finding fractions of shapes

Step 2: Compare and order unit fractions

Step 3: Compare and order fractions with the same denominator **R+R**

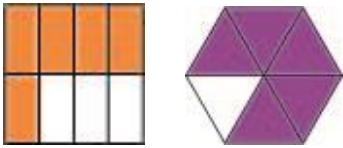
Step 4: Exploring equivalence

Step 5: Showing equivalence with accurate diagrams **R+R**

Destination Questions

1 

Identify the fraction shaded:


2 

Draw two different quadrilateral images to show the fraction:

$$\frac{8}{10}$$

3 

Order these unit fractions from largest to smallest.

$$\frac{1}{6}, \frac{1}{3}, \frac{1}{12}, \frac{1}{9}$$

Explain how you know.

4 

True or false? Explain why.

$$\frac{1}{8} > \frac{1}{6} \text{ because 8 is larger than 6.}$$

5 

Place these fractions on a blank number line.

$$\frac{1}{7}, \frac{4}{7}, \frac{2}{7}, \frac{6}{7}$$



Explain how you found your benchmarks.

6 

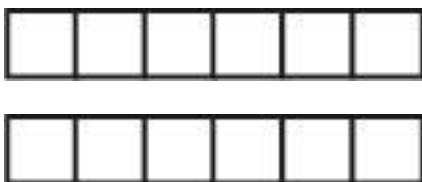
Order these fractions, with the same denominator, from largest to smallest.

$$\frac{2}{9}, \frac{6}{9}, \frac{4}{9}, \frac{1}{9}$$

7 

Use these diagrams to prove:

$$\frac{2}{6} = \frac{1}{3}$$


8 

Draw diagrams to show fractions equivalent to $\frac{1}{4}$.

9 

Complete the fractions below to make each number sentence true.

Draw a representation for each statement.

$$\frac{2}{3} = \frac{\quad}{6}$$

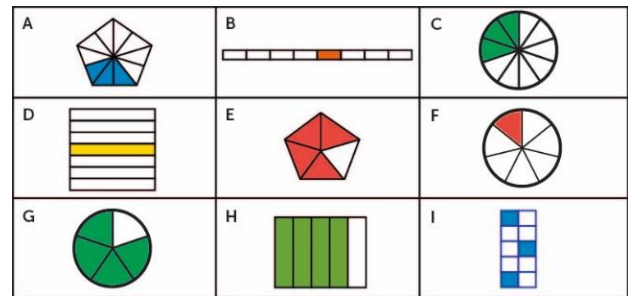
$$\frac{\quad}{10} = \frac{1}{2}$$

$$\frac{3}{\quad} = \frac{6}{8}$$

Step one

Finding fractions of shapes

Show the images from
handout_3LS22_step1_fractions_of_shapes.
Encourage pupils to use the language built from
3LS21 to describe the images shown.



Can you identify a unit fraction?

Image F shows a unit fraction
because the whole has been
divided into seven equal
parts and one part is shaded.

Can you identify a non-unit fraction?

Image H is a non-unit fraction. Four
of the five equal parts are shaded.

Reinforce that the denominator is the number of equal parts that the shape has been divided into and the numerator describes the number of equal parts shaded.

What do you notice
about shape I?

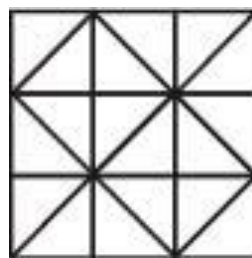
The denominator is ten because the shape has been
divided into tenths. The numerator is three as three of
the tenths are shaded. The fraction is written as $\frac{3}{10}$.

Pupils use handout_3LS22_step1_matching_fractions_of_shapes to become more familiar with identifying fractions of shape in a range of representations. Alongside this activity, pupils may use concrete resources to build the matching fraction of a quantity to reinforce the learning from 3LS21. For example, pupils could use counters or cubes to show the fractions from each shape on the handout.



Activities for exploring ideas at greater depth

Investigate how many triangles would need to be shaded to show: a) $\frac{1}{2}$, b) $\frac{2}{3}$, c) $\frac{1}{9}$



Amy says it is impossible to shade whole triangles in to show $\frac{1}{4}$. Is she correct? Explain why.

BUFFER ZONE

Step two

Compare and order unit fractions

Use questioning to reinforce the concept that a unit fraction is one part of a whole that is divided into equal parts. This means the numerator for a unit fraction (the number at the top of a fraction) is 1.

Use an example of a pizza for lunch, using the images from 3LS23_step2_order_unit_fractions_intro to support discussion. If the pizza was shared equally between 2 people, each person would receive one half each. This would be written as $\frac{1}{2}$. What if the same pizza was shared equally between 4 people? Or shared between 8 or 12?

What do you notice?

The smallest piece is $\frac{1}{12}$ because it has been cut into 12 equal pieces.

$\frac{1}{3}$ is larger than $\frac{1}{4}$ as less people are sharing. I have spotted that if the denominator is larger than the unit fraction piece gets smaller.

Pupils explore this practically using fraction equipment available or by folding paper of the same size. For example, folding large sticky notes into halves, quarters, eighths and noticing that whilst the denominator increases, the size of the fraction decreases.

Use handout_ 3LS22_step2_conversation_cartoon to discuss the common misconception that a unit fraction with a larger denominator is larger in size.

Who do you agree with? Explain why.

The boy has got confused as 12 is larger than 2 and 4. However, the $\frac{1}{12}$ is so much smaller than $\frac{1}{2}$ as it has been divided into 12 pieces rather than just two.

Pupils use game 3LS22_step2_order_unit_fractions_game to rehearse comparing fractions. Pupils record their pathway and justify their choices.

On the first step, I could not move from $\frac{1}{9}$ to $\frac{1}{12}$. Even though 12 is larger than 9, I know that $\frac{1}{12}$ is the smaller fraction as it has been divided into 12 equal pieces rather than 9. I have drawn images to show this.



3 

4 

Step three

Compare and order fractions with the same denominator



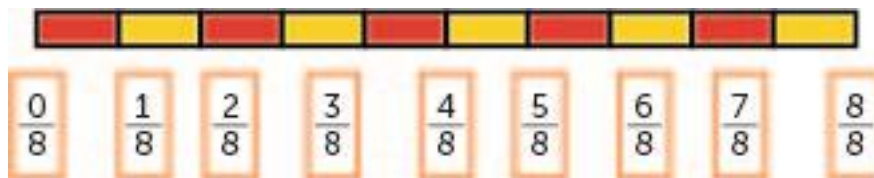
Use a counting stick as a 0-1 fractional number line. Use sticky notes to position fractions that have 10 as the denominator. Pupils should be becoming more fluent at counting up and down in tenths. Counting games such as the boomerang, hiccup and sneezing stick (used in 3LS16 step 4) could be used to provide further oral rehearsal.

Use the same stick to now locate, order and compare the fractions: $\frac{1}{8}$, $\frac{4}{8}$, $\frac{2}{8}$, $\frac{7}{8}$.

Use questioning to model how the eighths can be positioned on a counting stick without 8 equal sections marked. Draw the pupils' attention to how the numerator is used to order fractions with the same denominator.

$\frac{4}{8}$ is the equal to one half because 8 eighths is equal to a whole and half of 4 is 8. So $\frac{4}{8}$ must go in the middle.

I know $\frac{7}{8}$ is larger than $\frac{2}{8}$ because it is closer to a whole on the number line. Also, on a cake, if you cut it into eighths, 7 pieces is more cake than if you had 2 pieces.



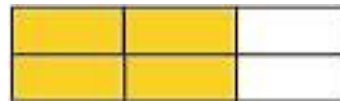
Pupils use the 'greater than' and 'less than' inequality symbols to compare fractions with the same denominator.

Pupils should be encouraged to use a pictorial representation alongside the written number sentence to justify their comparisons. For example:

$$\frac{3}{6} < \frac{4}{6}$$



<



$$\frac{3}{10} < \frac{6}{10} < \frac{9}{10}$$



Activities for exploring ideas at greater depth

Complete the missing numerators **using odd digits only**.

Which one is impossible to complete? Explain how you know.

$$\frac{3}{10} < \frac{\square}{10} < \frac{\square}{10}$$

$$\frac{\square}{10} < \frac{4}{10} < \frac{\square}{10}$$

$$\frac{\square}{10} < \frac{\square}{10} < \frac{3}{10}$$



Step four

Exploring equivalence

Use handout_3LS22_step4_equivalence_intro to revisit the equivalence introduced in year 2. First, encourage pupils to identify which fraction each bar represents and then make comparisons.

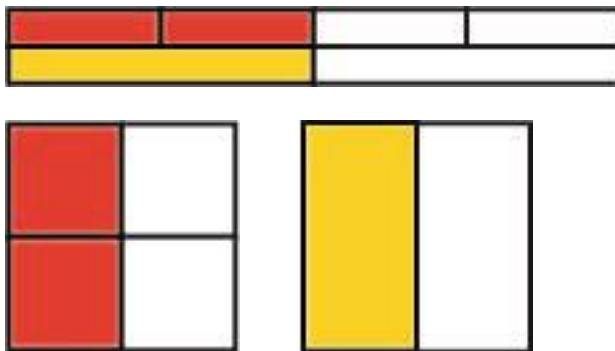
Which fractions can you identify?
What do you notice?

I know the yellow bar has been divided into four quarters as there are four equal pieces.

I've noticed that two purple bricks are equal to one orange bricks.
So two sixths are equal to one third. I can write this as $\frac{2}{6} = \frac{1}{3}$

In pairs, pupils explore equivalent fractions with small denominators using handout_3LS22_step4_equivalence_matching and concrete resources (either fraction cubes / bars if available or Cuisenaire rods).

Once pupils have had sufficient practical experience of these fractions, model how to draw different representations to show the same circle equivalent fractions as on handout_3LS22_step4_equivalence_matching.



I can divide each part into more equal parts. I divided each $\frac{1}{2}$ into 2 parts and now there are 4 parts altogether. There are $\frac{4}{4}$ in the whole and $\frac{2}{4}$ in the half.

I have re-drawn the model as bars to represent $\frac{1}{2} = \frac{2}{4}$

Pupils rehearse equivalence, using pictorial models, for other fractions with small denominators. For example:

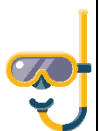
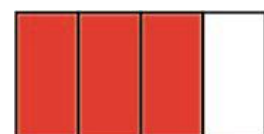
- $\frac{1}{3}$ and $\frac{2}{6}$
- $\frac{3}{4}$ and $\frac{4}{8}$
- $\frac{2}{3}$ and $\frac{4}{6}$
- $\frac{2}{5}$ and $\frac{4}{10}$



Activities for exploring ideas at greater depth

Which of these statements are true or false? Explain your reasoning.

- I can think of an equivalent fraction which has an odd digit as the numerator
- I can think of equivalent fractions in which one fraction has 12 as the numerator and one fraction has 12 as the denominator
- I can think of an equivalent fraction that has an odd digit as the denominator.



Step five



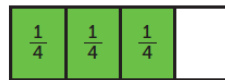
Showing equivalence with accurate diagrams

Use handout_3LS22_step5_conversation_cartoon to explore a common misconception.

3LS22

Step 5 Conversation Cartoon

Show three quarters as equal to six eighths.



Why do the diagrams not show that the fractions are equivalent?

The whole bars are not equal. It is not fair to compare the two.

I have noticed that $\frac{1}{4}$ and $\frac{1}{8}$ are drawn as the same size parts so you cannot compare the fractions.

Allow pupils to create their own correct diagrams for the problem. Share different examples as a class to reiterate that the bars must be the same size before they are divided into fractional pieces.

Pupils create their own images to show equivalent fractions such as:

$$\frac{4}{8} = \frac{1}{2}$$

$$\frac{2}{3} = \frac{4}{6}$$

$$\frac{1}{5} = \frac{2}{10}$$

Hands on resources and the use of fraction walls should be used to support pupils to access the learning and make links between the concepts across this sequence.



Activities for exploring ideas at greater depth

Using what you have noticed about the relationship between the numerator and denominators in equivalent fractions, can you complete these missing numbers? Convince me.

$$\frac{\square}{20} = \frac{1}{2}$$

$$\frac{2}{3} = \frac{\square}{36}$$

$$\frac{3}{4} = \frac{18}{\square}$$

